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Book Review: *Markov Processes, Characterization and Convergence*

Markov Processes, Characterization and Convergence. Stewart N. Ethier and Thomas G. Kurtz, Wiley, New York, 1986.

As Ethier and Kurtz tell us in their preface, their original aims were to treat the general problem of weak convergence of a sequence of Markov processes to a Markov limit. Such problems are not only not new, but are classical. Who does not know of the convergence of a random walk to Brownian motion (as the number of steps becomes infinite and time and space are appropriately normalized) or of continuous-time diffusion approximations to branching-type processes in discrete time? Not only are these results themselves now classical, but so were the majority of the techniques (most of which were not terribly elegant) that were originally used in their derivation. In recent years, however, this rather stable area of stochastic process theory has undergone a dramatic facelift, in view of substantial progress in both the areas of Markov processes and weak convergence. This book is an excellent account of this progress, told by authors who themselves have contributed significantly to its development.

The first step in any treatment of Markov processes must be to provide techniques for their characterization. Three related but basically different approaches are available. The first, the most classical, is the essentially functional-analytic semigroup approach of identifying Markov processes with associated infinitesimal generators. The second, the most modern, is of using the martingale approach initiated by Strook and Varadhan. This is an approach considerably more probabilistic in nature than semigroup theory, and very often much more natural to use. The third approach, which is generally of more limited scope, but often provides more insight when it is available, is that of treating specific Markov processes as solutions of stochastic differential equations.

Given the above three methods of characterization, three corresponding approaches are available to study the convergence (in distribution) of a sequence $X_1(t)$, $X_2(t)$,..., of Markov processes to a limit Markov process $X_\infty(t)$. The first develops the argument that if A_1, A_2,..., is

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the corresponding sequence of infinitesimal generators, and these converge, as operators, to the infinitesimal generator corresponding to X_∞, then one should be able to conclude that the X_n converge to X_∞ as $n \to \infty$. The second approach follows the line that if the X_n are a sequence of solutions of a corresponding sequence of martingale problems, and, in an appropriate sense, these problems converge to one whose solution is X_∞, then the same conclusion should hold. Similarly, if the X_n are solutions of a sequence of stochastic differential equations whose limiting form as $n \to \infty$ has X_∞ as a solution, we should be able to conclude that the X_n converge to X_∞.

Filling in the details of the sketchy outline just given takes about 300 pages, from which it should be obvious that these are not simple problems. Nevertheless, throughout their treatment of a general theory of Markov processes and weak convergence, via three quite distinct routes, Ethier and Kurtz firmly take the hand of the reader and lead him or her eagerly to the intrinsically interesting results, with gentle prodding through the necessary but often uninteresting details, and careful help over the technically difficult sections. As a result they have produced a book which, although it was originally designed as a research tract, serves as excellent source from which to learn about Markov processes in general, even if one does not have a specific interest in weak convergence problems.

The last 200 pages of the book (modulo a short technical appendix) are made up of a variety of examples of particular classes of Markov processes to which the general theory of the earlier chapters can be applied. There are chapters on branching processes, genetic models, population processes, and random evolutions. Included in these examples is a large number of specific examples coming from a variety of different disciplines.

All told, Ethier and Kurtz have produced an excellent treatment of the modern theory of Markov processses that will be useful both as a reference work and as a graduate textbook (exercises and teaching plans are included), although it is not a book to be recommended to weak students. Given the number of references that have already appeared to the prepublication versions of this book in the technical literature, it is probably already on its way to becoming a classic in its area.

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Book Review: *Noise in Physical Systems and 1/f Noise*

Noise in Physical Systems and 1/f Noise. A. d'Amico and P. Mazzetti, eds., North-Holland, Amsterdam, 1986.

This book is a collection of brief reports and somewhat longer invited contributions representing the proceedings of the 8th International Conference on Noise in Physical Systems and the 4th International Conference on $1/f$ Noise, held in Rome, September 9–13, 1985. As the title implies, the conference was divided into two broad categories, each consisting of theoretical and experimental contributions. In this book there are 11 invited papers, ranging over topics in the theory of and experiments on $1/f$ noise, through simulations of biological membrane noise, to chaos in nonlinear electronic circuits and the very interesting measurements of squeezed noise in transducers used on gravitational radiation detectors. These are followed by 100 brief accounts of the contributed talks on topics grouped under the headings: Theory, Devices, Biological Systems; Various Systems, Quantum Noise, Measurement Techniques, and Miscellaneous.

As is true of most conference proceedings, one cannot hope to use the accounts for tutorial purposes, but instead to find out *who* is doing *what*, along with, hopefully, a good set of references. Though a range of stimulating and creative activity is reported, a number of important names, especially in the sections on $1/f$ noise, are conspicuously absent, so that the reader will have to search elsewhere to complete her or his knowledge of *who*. Though some exciting and very contemporary topics are explored in the invited papers, one might wish for a more nearly "review type" style in others, whose references are limited, so that additional searches must be made to find out *what* as well. Over half the invited papers were authored or coauthored by members of the International Programme Committee. As is true of most larger conferences, the quality and credibility of the papers in several categories span a wide range, even though, as stated in the Preface, *all* manuscripts were refereed by the International Programme Committee. Notable for their high current interest and readability are the contributed papers on macroscopic quantum tunneling and the experimental observations of deterministic noise (chaos) in photoconductors.

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In spite of the defects, the reader interested in noise in physical systems will find a number of new, interesting, and useful selections here, and she or he will find the subject and author indices helpful.

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